Name: Feedback

SA367 · Mathematical Models for Decision Making Spring 2022 · Uhan

Exam 1 – 2/10/2022

Instructions

- You have until the end of the class period to complete this exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

Problem	Weight	Score
1	3	
2a	0.5	
2b	0.5	
3	3	
4a	0.5	
4b	0.5	
5	3	
6a	0.5	
6b	0.5	
7	2	
Total		/ 140

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

> The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Problem 1. The Primal Pasta Company is developing a replacement policy for its large industrial mixer. The company wants a policy that covers a 5 year period. At the beginning of the first year, the company must purchase a new mixer. At the beginning of each subsequent year, the company can keep the mixer or replace it. The mixer must be in service for at least 1 year and no more than 4 years. A new mixer costs $25,000. The cost of maintaining the mixer and the resale value at the end of a year depend on the age of the mixer:

	Age of mixer at the beginning of the year			
	0 years	1 years	2 years	3 years
Maintenance costs ($)	1,000	2,000	3,000	3,000
Resale value at the end of the year ($)	15,000	10,000	8,000	7,000

The company's goal is to minimize the total cost of having a mixer over a 5 year period. Formulate this problem as a shortest path problem. In particular:

- draw the directed graph (nodes and edges),
- specify the edge lengths, and
- specify the source and sink nodes.

Most of you had the correct directed graph, source node, and sink node.

Note that the maintenance costs given depend on the age of the mixer at the beginning of every year the mixer is in service.

For similar problems, see Example 4 in Lesson 1, as well as Problems 1 and 2 in Lesson 1 (page 9), assigned for homework.

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Problem 2. Suppose you solved the shortest path problem you formulated in Problem 1 with the Bellman-Ford algorithm. The algorithm outputs (i) the length of a shortest path, and (ii) the edges in a shortest path.

a. Briefly explain how you would use this output to determine the minimum total cost of having a mixer over a 5 year period. Give a hypothetical example if it helps.

This is for both parts a and b:

Some of you solved the shortest path problem you formulated in Problem 1. Note that this isn't necessary.

The objective of this problem is for you to explain how to use the output of the Bellman-Ford algorithm when applied to the shortest path problem you formulated in Problem 1.

Recall from Lesson 2: the Bellman-Ford algorithm solves a shortest path problem and gives you two outputs: (i) the length of a shortest path, and (ii) the edges (or nodes) in a shortest path.

If you're struggling with explaining how to use these outputs, look at the hints given in the Quiz 1 feedback on the course website. Also, look at the end of the solutions to Problems 1 and 2 in Lesson 1, assigned for homework.

b. Briefly explain how you would use this output to determine the corresponding replacement policy for the mixer. Give a hypothetical example if it helps.

Problem 3. The Riemann Real Estate Company owns a short-term rental property with several interested potential renters for the next 8 weeks. Here are the bids offered by these potential renters:

Renter	Starting week	Duration (weeks)	Bid ($)
1	1	2	1000
2	2	1	700
3	2	3	1500
4	4	1	600
5	4	3	1800
6	6	2	1400
7	7	2	1000

The starting weeks are numbered sequentially 1 through 8. At most one renter can occupy the property at any time. In particular, the company has the option of leaving the property unoccupied during any given week. The company's goal is to maximize the total revenue earned from the rental property over the next 8 weeks.

Formulate the company's problem as a shortest path problem. In particular:

- draw the directed graph (nodes and edges),
- specify the edge lengths, and
- specify the source and sink nodes.

- Almost all of you drew a directed graph with nodes corresponding to the beginning of weeks 1 through 9, covering the 8-week time horizon described in the problem. This is the right idea.
- Many of you only included edges corresponding to the 7 potential renters. In such a graph, there is no path between nodes 1 and 9! This is why you need to explicitly model the option to leave the property unoccupied.
- Note that this problem asks you to maximize the total revenue earned from the rental property, but you are modeling the setting as a shortest path problem. Be careful with how you set up the edge lengths.

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Problem 4. Suppose you solved the shortest path problem you formulated in Problem 3 with the Bellman-Ford algorithm. The algorithm outputs (i) the length of a shortest path, and (ii) the edges in a shortest path.

a. Briefly explain how you would use this output to determine the maximum total revenue that the company can earn from the rental property over the next 8 weeks. Give a hypothetical example if it helps.

For parts a and b: see the feedback for Problem 2 for some hints if you're struggling with explaining how to use the outputs of the Bellman-Ford algorithm.

b. Briefly explain how you would use this output to determine which renters the company should select in order to maximize its total revenue from the rental property over the next 8 weeks. Give a hypothetical example if it helps.

Problem 5. Professor May B. Wright is planning to go Bali, Indonesia after the semester ends. She needs to put together an itinerary from Washington, DC (IAD) to Jakarta (CGK), and is considering the following flights:

Flight No.	Origin	Destination	Departure Time	Arrival Time	Flight Time (hours)
1	IAD	DFW	5:00	8:00	4
2	IAD	DFW	8:00	11:00	4
3	IAD	NRT	7:00	11:00	15
4	DFW	NRT	9:00	14:00	14
5	DFW	NRT	12:00	17:00	14
6	NRT	CGK	15:00	21:00	8
7	NRT	CGK	18:00	24:00	8

In the table above, the departure times are local at the origin airport, and the arrival times are local at the destination airport. Professor Wright would like to find an itinerary from IAD to CGK with the minimum total travel time; that is, the minimum total flight time plus layover time. Assume that the minimum layover time between flights is 45 minutes.

Formulate Professor Wright's problem as a shortest path problem. In particular:

- draw the directed graph (nodes and edges),
- specify the edge lengths, and
- specify the source and sink nodes.

Make sure your model includes all possible itineraries.

- Many of you had the right idea.
- If you're not sure how to start, take a look at how we modeled the mileage running problem as a shortest path problem at the beginning of Lesson 4. In that model, we represented the different flights as nodes.

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Problem 6. Suppose you solved the shortest path problem you formulated in Problem 5 with the Bellman-Ford algorithm. The algorithm outputs (i) the length of a shortest path, and (ii) the edges in a shortest path.

a. Briefly explain how you would use this output to determine the minimum total travel time from IAD to CGK. Give a hypothetical example if it helps.

For parts a and b: see the feedback for Problem 2 for some hints if you're struggling with explaining how to use the outputs of the Bellman-Ford algorithm.

b. Briefly explain how you would use this output to determine an itinerary (i.e., sequence of flights) that minimizes the total travel time from IAD to CGK. Give a hypothetical example if it helps.

Problem 7. Describe the shortest path problem being solved by the code below. In particular:

- draw the directed graph (nodes and edges),
- specify the edge lengths, and
- specify the source and sink nodes.

```
1  import networkx as nx
2  import bellmanford as bf
3
4  cost = {1: 11, 2: 18, 3: 23, 4: 27}
5
6  G = nx.DiGraph()
7
8  for i in range(1, 7):
9      G.add_node(i)
10
11 for i in range(1, 7):
12     for j in range(1, 5):
13         if i + j < 7:
14             G.add_edge(i, i + j, length=cost[j])
15
16 length, nodes, negative_cycle = bf.bellman_ford(G, source=1, target=6, weight="length")
```

- Most of you had the right idea here.
- Note that the dictionary `cost` used in line 14 is defined in line 4.
- If you struggled with this problem, take a look at the solutions to Problem 1 in Lesson 4, assigned for homework, for a similar problem.